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$\mu - e$ conversion in nuclei in the left-right supersymmetric model

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Abstract. We investigate the contributions to $\mu - e$ conversion in nuclei in a supersymmetric model with left-right symmetry, motivated by the new data on neutrino oscillations. We study the dependence of the conversion rate on the various parameters of the model, and show that light-mass or large $\tan\beta$ scenarios are severely restricted. We analyse the effect of several popular mecahnisms of neutrino mixing on the conversion rate as well as the influence of the right-handed scale on the conversion rate. We compare the conversion rate to the branching ratio for $\mu \to e\gamma$ and discuss their relative accessibility at future experiments, their sensitivity to various parameters of the model, as well as their relative importance in providing signals for new Physics.

1 Introduction

The indications for neutrino masses and oscillations coming principally from the Super-Kamiokande [1] data on atmospheric neutrinos has given a boost to Physics beyond the standard model on one hand, and lepton flavour violation on the other hand. In the minimal standard model (SM) neutrino masses vanish and lepton flavour is conserved separately for each generation. This is no longer true if new particles and/or interactions are introduced. Since both the solar and the atmospheric neutrino data can be accomodated in a natural way in schemes with three or four light neutrinos and mixing between them (the details of which are still under debate), there is a clear expectation in all schemes that individual lepton numbers will be violated. The prospect for measuring charged-lepton number violation is very good, the present upper limits on the most interesting ones are:

$$BR(\mu^+ \to e^+\gamma) < 1.2 \times 10^{-11} \text{ [2]} \tag{1}$$

$$BR(\mu^+ \to e^+e^+e^-) < 1.0 \times 10^{-12} \text{ [3]} \tag{2}$$

$$R(\mu^- Ti \to e^- Ti) < < 6.1 \times 10^{-13} \text{ [4]} \tag{3}$$

$$BR(\tau^+ \to \mu^+\gamma) < 1.1 \times 10^{-6} \text{ [5]} \tag{4}$$

Projects are currently underway to improve several of these limits significantly. Of these processes, $\mu - e$ conversion in nuclei is perhaps the most interesting experimentally. From a theoretical point of view, it is the most difficult to disentangle, because of the interdependence between particle and nuclear physics elements, in particular the difficulty in evaluating nuclear matrix elements [6]. But the process is very interesting because it has quite a different structure from $\mu \to e\gamma$ (as opposed to $\mu^+ \to e^+e^+e^-$). Therefore it provides complimentary information on muon decay from the first two decays: it can occur even when $\mu \to e\gamma$ is forbidden, and it could be a better indicator of a rich gauge structure, such as extra Z or W bosons [7], or extra Higgs/Higgsinos [8].

Among theoretical models which predict observable lepton flavour violation, supersymmetric models (SUSY) have received special attention. In SUSY models, lepton flavour could be violated at very high scales such as the grand unification scale or the mass scale of a heavy right-handed neutrino. Lepton flavour violation would then be an indication for physics at high scales. Supersymmetry has many theoretically attractive features, such as explaining the boson-fermion symmetry and also providing a mechanism for solving two of the fundamental problems in the Standard Model: stability under radiative corrections and the origin of the elecro-weak scale. The most popular realization of supersymmetry, the Minimal Supersymmetric Standard Model (MSSM), while filling in some of the theoretical gaps of the Standard Model, fails to explain other phenomena such as the weak mixing angle, the small mass of the known neutrinos, the origin of CP violation, or the absence of rapid proton decay. Extended gauge structures such as supersymmetric grand unified theories, introduced to provide an elegant framework for the unification of forces [9], would connect the standard model with more fundamental structures such as superstrings, and also would resolve the puzzles of the electroweak theory. Among these $SU(5)$ and $SO(10)$ have been the most popular realizations of GUT scenarios. With the advent of neutrino masses, the evidence for grand unification is very strong. Previous pieces of the puzzle were mosly circumstantial: the observed family structure and intriguiging features of the fermion masses and mixings, the unification of the gauge couplings at $M_{GUT} \approx 2 \times 10^{16} GeV$, baryogenesis and the explanation for dark matter. It has been argued that all of these point towards not just grand unification, but select a particular scheme for such unifi-

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cation, based on supersymmetry and left-right symmetry [10]. If one augments these by $SU(4)_c$, the theory provides strong predictions for quark and lepton masses, as an effective string-unified G(224) or SO(10) symmetry. Building realistic brane world from Type I strings also involves a left-right supersymmetric theory with supersymmetry broken at the string scale, $M_{SUSY} \approx 10^{10-12}\ GeV$ [11]. Such theories have the added benefit of a stable proton, natural R stringy symmetry and gauge unification. If instead $M_{SUSY} \approx 1\,TeV$, all of the features except for gauge unification are maintained. The advantages of combining left-right symmetry with supersymmetry (LR SUSY) have long been known [12–15]. LR SUSY was originally seen as a natural way to suppress rapid proton decay and as a mechanism for providing small neutrino masses through the see-saw mechanism [14]. Later it has been shown that it could offer a solution to both the strong and the weak CP problem [16].

The decay $\mu \to e\gamma$, a sensitive probe of physics at the Planck scale, has been extensively studied in the context of LR SUSY [17]. It has been shown that the LR SUSY model is capable of giving rise to large lepton-flavour decay rates, in much the same way as SO(10) [18,19], through potentially large Yukawa couplings for the neutrino, h_ν. It was also shown to be enhanced compared to the corresponding MSSM rate. For completness, we study here the $\mu - e$ conversion and compare it to the $\mu \to e\gamma$ decay. We are interested in the differences between the leptonic matrix elements of the two, and in comparing the parameter space that can be best analysed with one or the other muon decay. We also investigate the dependence of the conversion rate on several popular scenarios for (s)neutrino mixings. We study the non-photonic contributions and show the dependence of the muon conversion rate on the right-handed scale of the theory, and discuss possibilities of restricting the parameter space for detection of right-handed particles. We show that, in the case of expected improved experimental accuracy, muon to electron conversion in nuclei will likely become the premier place to study muon flavour violation.

Our paper is organized as follows: We discuss the LR SUSY model in Sect. 2. Sources of flavour violation through slepton/sneutrino mixings are discussed in Sect. 3. We analyse the photonic contributions to muon conversion in Sect. 4.1 and the non-photonic one in Sect. 4.2. After the numerical analysis and discussion in Sect. 5, we conclude in Sect. 6.

2 The left-right supersymmetric model

One might consider the left-right supersymmetric model as either a natural extension of the minimal supersymmetric model (MSSM) or as an intermediate gauge structure from the breaking of a supersymmetric GUT-favored model, such as $SO(10)$ or E_6. The LR SUSY model, based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, has matter doublets for both left- and right- handed fermions and the corresponding left- and right-handed scalar partners (sleptons and squarks) [20]. In the gauge sector, corresponding to $SU(2)_L$ and $SU(2)_R$, there are triplet gauge bosons $(W^{+,-}, W^0)_L$, $(W^{+,-}, W^0)_R$, and a singlet gauge boson V corresponding to $U(1)_{B-L}$, together with their superpartners. The Higgs sector of this model consists of two Higgs bi-doublets, $\Phi_u(\frac{1}{2}, \frac{1}{2}, 0)$ and $\Phi_d(\frac{1}{2}, \frac{1}{2}, 0)$, which are required to give masses to both the up and down quarks. The phenomenology of the doublet Higgs is similar to the non-supersymmetric left-right model [13], except that the second pair of Higgs doublet fields, which provide new contributions to the flavour-changing neutral currents, must be heavy, in the 5-10 TeV range, effectively decoupling from the low-energy spectrum [21]. The spontaneous symmetry breaking of the group $SU(2)_R \times U(1)_{B-L}$ to the hypercharge symmetry group $U(1)_Y$ is accomplished by the vacuum expectation values of a pair of Higgs triplet fields $\Delta_L(1, 0, 2)$ and $\Delta_R(0, 1, 2)$, which transform as the adjoint representation of $SU(2)_R$. The choice of the triplets (versus four doublets) is preferred because with this choice a large Majorana mass can be generated (through the see-saw mechanism) for the right-handed neutrino and a small one for the left-handed neutrino [13]. In addition to the triplets $\Delta_{L,R}$, the model must contain two additional triplets $\delta_L(1, 0, -2)$ and $\delta_R(0, 1, -2)$, with quantum number $B - L = -2$ to insure cancellation of the anomalies which would otherwise occur in the fermionic sector. Given their strange quantum numbers, the δ_L and δ_R do not couple to any of the particles in the theory, so their contribution is negligible for any phenomenological studies.

The superpotential for the LR SUSY is

$$\begin{aligned} W = & \mathbf{h}_q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q^c + \mathbf{h}_l^{(i)} L^T \tau_2 \Phi_i \tau_2 L^c \\ & + i(\mathbf{h}_{LR} L^T \tau_2 \Delta L + \mathbf{h}_{LR} L^{cT} \tau_2 \Delta^c L^c) \\ & + M_{LR} \left[Tr(\Delta \bar{\Delta} + Tr(\Delta^c \bar{\Delta}^c) \right] \\ & + \mu_{ij} Tr(\tau_2 \Phi_i^T \tau_2 \Phi_j) + W_{NR} \end{aligned} \quad (5)$$

where W_{NR} denotes (possible) non-renormalizable terms arising from higher scale physics or Planck scale effects [22]. The presence of these terms insures that, when the SUSY breaking scale is above M_{W_R}, the ground state is R-parity conserving [23].

As in the standard model, in order to preserve $U(1)_{EM}$ gauge invariance, only the neutral Higgs fields acquire non-zero vacuum expectation values ($VEV's$). These values are:

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \langle\ \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \text{ and}$$
$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\omega} \end{pmatrix}.$$

$\langle \Phi \rangle$ causes the mixing of W_L and W_R bosons with CP-violating phase ω. In order to simplify, we will take the $VEV's$ of the Higgs fields as: $\langle \Delta_L \rangle = 0$ and

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \langle \Phi_u \rangle = \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \langle \Phi_{\rm d} \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}.$$

Choosing $v_L = \kappa' = 0$ satisfies the more loosely required hierarchy $v_R \gg max(\kappa, \kappa') \gg v_L$ and also the required cancellation of flavour-changing neutral currents. The Higgs fields acquire non-zero $VEV's$ to break both parity and $SU(2)_R$. In the first stage of breaking, the right-handed gauge bosons, W_R and Z_R acquire masses proportional to v_R and become much heavier than the usual (left-handed) neutral gauge bosons W_L and Z_L, which pick up masses proportional to κ_u and κ_d at the second stage of breaking.

The supersymmetric sector of the model, while preserving left-right symmetry, has eight singly-charged charginos, corresponding to $\tilde{\lambda}_L, \tilde{\lambda}_R, \tilde{\phi}_u$, $\tilde{\phi}_d$, $\tilde{\Delta}_L^-$, $\tilde{\Delta}_R^-$, $\tilde{\delta}_L^-$ and $\tilde{\delta}_R^-$. The model also has eleven neutralinos, corresponding to $\tilde{\lambda}_Z$, $\tilde{\lambda}_{Z'}$, $\tilde{\lambda}_V$, $\tilde{\phi}_{1u}^0$,$\tilde{\phi}_{2u}^0$, $\tilde{\phi}_{1d}^0$, $\tilde{\phi}_{2d}^0$, $\tilde{\Delta}_L^0$, $\tilde{\Delta}_R^0$ $\tilde{\delta}_L^0$, and $\tilde{\delta}_R^0$. It has been shown that in the scalar sector, the left-triplet Δ_L couplings can be neglected in phenomenological analyses of muon and tau decays [24]. Although Δ_L is not necessary for symmetry breaking [15] and is introduced only for preserving left-right symmetry, both Δ_L^{--} and its right-handed counterpart Δ_R^{--} play very important roles in phenomenological studies of the LR SUSY model. It has been shown that these bosons, and possibly their fermionic counterparts, are light [22]. Also, these doubly charged Higgs and their corresponding Higgsinos lead to an enhancement in lepton-flavour violating decays [8].

3 Flavour violation in LR SUSY: slepton and sneutrino mixing

The sources of flavour violation in the LR SUSY model come from either the Yukawa potential or from the trilinear scalar coupling.

The interaction of fermions with scalar (Higgs) fields has the following form:

$$\begin{aligned}\mathcal{L}_Y &= \mathbf{h}_u \bar{Q}_L \Phi_u Q_R + \mathbf{h}_d \bar{Q}_L \Phi_d Q_R + \mathbf{h}_\nu \bar{L}_L \Phi_u L_R \\ &\quad + \mathbf{h}_e \bar{L}_L \Phi_d L_R + H.c.; \\ \mathcal{L}_M &= i\mathbf{h}_{LR}(L_L^T C^{-1} \tau_2 \Delta_L L_L + L_R^T C^{-1} \tau_2 \Delta_R L_R) \\ &\quad + H.c. \end{aligned} \tag{6}$$

where $\mathbf{h}_u$, $\mathbf{h}_d$, $\mathbf{h}_\nu$ and $\mathbf{h}_e$ are the Yukawa couplings for the up and down quarks and neutrino and electron, respectively, and $\mathbf{h}_{LR}$ is the coupling for the triplet Higgs bosons. LR symmetry requires all $\mathbf{h}$-matrices to be Hermitean in the generation space and $\mathbf{h}_{LR}$ matrix to be symmetric. The Yukawa matrices have physical and geometrical significance and cannot be rotated away. Geometrically, they represent misallignement between the particle and sparticle bases in flavour space. Their physical significance is that they cause flavour violation. The triliniear scalar couplings appear in the soft-scalar mass term in the Lagrangian:

$$\begin{aligned}\mathcal{L}_{soft} &= -\Big[\mathbf{A}_q^i \mathbf{h}_q^{(i)} \tilde{Q}^T \tau_2 \Phi_i \tau_2 \tilde{Q}^c + \mathbf{A}_l^i \mathbf{h}_l^{(i)} \tilde{L}^T \tau_2 \Phi_i \tau_2 \tilde{L}^c \\ &\quad + i\mathbf{A}_{LR}^i \mathbf{h}_{LR}(\tilde{L}^T \tau_2 \Delta L + L^{cT} \tau_2 \Delta^c \tilde{L}^c)\Big] \\ &\quad - \Big[M_L \tilde{W}_L \tilde{W}_L + M_R \tilde{W}_R \tilde{W}_R + M_V \tilde{V} \tilde{V}\Big] \\ &\quad - M_\Delta^2 \left[Tr(\Delta \bar{\Delta} + Tr(\Delta^c \bar{\Delta}^c)\right] \\ &\quad - B\mu_{ij} \Phi_i \Phi_j - \mu_{ij}^2 \Phi_i \Phi_j \end{aligned} \tag{7}$$

where the $\mathbf{A}$-matrices (A_u, A_d, A_ν and A_e) are of a similar form to the Yukawa couplings and provide additional sources of flavour violation; and B is a mass term. For simplicity, and to avoid violating the bound on $\mu \to e\gamma$, we shall assume a universal form of supersymmetry breaking, namely a universal scalar mass m_0 and a trilinear scalar coupling with a universal parameter $\mathbf{A}$ at the GUT scale. The Dirac neutrino and the charged lepton Yukawa couplings cannot, in general, be diagonalized simultaneously, which means that the lepton Yukawa and slepton mass matrices cannot be diagonalized simultaneously either [25].

The charged slepton masses are eigenvalues of the 6×6 matrix originating from the Lagrangian:

$$\begin{aligned}\mathcal{L}_{M_{\tilde{l}}} &= \begin{pmatrix} \tilde{l}_L^\dagger & \tilde{l}_R^\dagger \end{pmatrix} \\ &\quad \times \begin{pmatrix} \mu_L^2 + m_l^2 + \delta m_{\nu_D}^2 + c_\nu h_\nu^2 & \mathcal{A}_l^* m_l \\ \mathcal{A}_l m_l & \mu_R^2 + m_l^2 + c_\nu h_\nu^2 \end{pmatrix} \\ &\quad \times \begin{pmatrix} \tilde{l}_L \\ \tilde{l}_R \end{pmatrix}, \end{aligned} \tag{8}$$

where $\tan\beta = \kappa_d/\kappa_u$ is the ratio of the vacuum expectation values of the bidoublet Higgs, $\mathcal{A}_l \sim A_l + \delta A_l + \mu \tan\beta$, μ_L^2, μ_R^2 and A_l are diagonal contributions to the corresponding matrices and $\delta m_{\nu_D}^2$, δA_l are off-diagonal terms which appear because h_ν and h_l may not be diagonalized simultaneously. All the entries are 3×3 in flavour space. The difference beween the LR SUSY model and the MSSM is in the appearance of the terms proportional to the neutrino Yukawas. These terms appear either at the tree level or at the one-loop renormalization from M_{GUT} [17]. If LR symmetry is conserved, the masses of the left and right sleptons are equal. After breaking, the mass difference between the scalar partners of the left handed and right handed leptons is much larger than the mass difference between the generations. We expect $\frac{m_{\tilde{l}_1}^2 - m_{\tilde{l}_2}^2}{m_{\tilde{l}}^2} \sim 10^{-2} - 10^{-1}$ [16,21]. The full mass for left- and right-handed sneutrino has a complicated 12×12 matrix structure. We can construct however an effective 6×6 matrix for the light neutrinos using the sea-saw mechanism:

$$(m_{\tilde{\nu}}^2)_{eff} = \begin{pmatrix} \mu_L'^2 + m_\nu^2 + c_l h_l^2 & \mathcal{A}_\nu^*(m_D M^{-1} m_D^\dagger) \\ \mathcal{A}_\nu(m_D M^{-1} m_D^\dagger) & \mu_R'^2 + m_\nu^2 + c_l h_l^2 \end{pmatrix} \tag{9}$$

where $\mathcal{A}_\nu \sim 2A_\nu + A_N + 2\mu\cot\beta$. Note that in LR model, the left-handed neutrino mass is allowed to be nonzero, but can be made small through the see-saw mechanism, as long as the right-handed neutrino is very heavy (masses of order 1 TeV or so are consistent with the upper limit on the right-handed electron neutrino mass [12]). Despite the presence of the two scalar neutrinos, the mixing between the right-handed and the left-handed sneutrinos is small, due to the see-saw mechanism in the sfermion sector. The left-right elements of the sneutrino mass matrix are proportional to the Dirac neutrino mass, which can be significant. But the right-right element of the sneutrino mass matrix is very heavy, so the mixing of sneutrino will be supressed by the inverse M_R^2. However, as opposed to the charged sleptons, in the light sneutrino sector the Dirac terms do not induce considerable mixing [25]. The off-diagonal terms in the sneutrino mass matrix mix degenerate states and do not affect flavour violating decays.

Next we consider the implications of these flavour changing mechanisms in LRSUSY on $\mu - e$ conversion in nuclei.

4 The amplitude for the process $\mu - e$

All the mechanisms for $\mu - e$ conversion found in the literature fall into two categories: photonic and non-photonic. These classes differ from the point of view of the nucleon and nuclear structure calculation. We concentrate here on the leptonic contributions, and quote results for the nucleonic structure from previous considerations in the literature.

The photonic mechanisms are mediated by the photon exchange between the quark and the $\mu - e$ lepton currents. They are induced at one-loop level by the lepton-flavour non-diagonal vertex. The mechanisms is similar to the one in the radiative $\mu \to e\gamma$ decay, except that the photon emission can also be off shell. This is an important difference and produces rather different transition amplitudes for the two processes.

The non-photonic mechanism contains heavy particles in intermediate states and can also occur at tree level through a Z_L or Z_R penguin, or through the box diagram. The non-photonic contributions are in general smaller than the photonic ones, and this will also be the case here. However, due to their importance in constraining the right-handed mass, we will include a full analysis here. The Feynman diagrams which contribute to the $\mu - e$ conversion in the LR SUSY model are given in Fig. 1. The amplitude for the $\mu - e$ can be writen in the general form:

$$\mathcal{M}_{\mu-e} = \frac{4\pi\alpha}{q^2} J_\lambda^{(1)} j_{(1)}^\lambda + \frac{g'}{m_\mu^2} J_\lambda^{(2)} j_{(2)}^\lambda \tag{10}$$

where the first term is the photonic contribution and the second the non-photonic contribution. Here g' is the coupling for Z_L or Z_R and q is the momentum transfer. We denote by J hadronic and j leptonic currents. We analyse the photonic and non-photonic contributions in detail below.

4.1 The photonic contributions

The first terms corresponding to the photonic contribution in Fig. 1 (a and d) can be written in detail as follows. For the hadronic currents:

$$J_\lambda^{(1)} = \bar{N}\gamma_\lambda \frac{1+\tau_3}{2} N \tag{11}$$

and for the leptonic current:

$$j_{(1)}^\lambda = \bar{u}(p)\left[(f_{M1} + \gamma_5 f_{E1}) i\sigma^{\lambda\nu} \frac{q_\nu}{m_\mu} + (f_{E0} + \gamma_5 f_{M0}) i\gamma^\nu \left(g_{\lambda\nu} - \frac{q^\lambda q^\nu}{q^2}\right)\right] u(p+q) \tag{12}$$

The functions f_{M0}, f_{M1}, f_{E0}, f_{E1} are the electromagnetic form factors and are functions of q^2. For the decay $\mu \to e\gamma$ only $f_{M1}(0)$ and $f_{E1}(0)$ can contribute, whereas all four contribute for the $\mu - e$ conversion.

Based on the above transition elements, the branching ratio for the coherent $\mu^- - e^-$ conversion is given by [26]:

$$\begin{aligned} R_{ph}(\mu^- N \to e^- N) &= \frac{p_e E_e Z \alpha^5 Z_{eff}^4 F_p^2}{m_\mu \Gamma_{capt}} \Big\{ |f_{E0}(-m_\mu^2) + f_{M1}(-m_\mu^2) \\ &+ f_{M0}(-m_\mu^2) + f_{E1}(-m_\mu^2)|^2 + |f_{E0}(-m_\mu^2) \\ &+ f_{M1}(-m_\mu^2) - f_{M0}(-m_\mu^2) - f_{E1}(-m_\mu^2)|^2 \Big\} \end{aligned} \tag{13}$$

where Γ_{capt} is the total muon capture rate, Z_{eff} is an effective atomic charge obtained by averaging the muon wave function over the nuclear density, and F_p is the nuclear matrix element. The $\mu - e$ conversion process is roughly proportional to $Z_{eff}^4 Z$, while the normal muon capture Γ_{capt} is proportional to Z_{eff}^4, resulting in an enhancenment of the conversion rate by a factor of Z [27].

The contributions to the conversion $\mu \to e$ in the Left-Right Supersymmetric Model are parametrized by the elements of the mass matrices: the Yukawa matrix, the left-handed and the right-handed lepton doublet scalar mass matrix and the sneutrino mass matrix. The associated physical parameters are the Yukawa eigenvalues; the mass eigenvalues for the left- and right-handed slepton doublets; and mixing angles for both left lepton doublets, θ_L, and right handed lepton doublets, θ_R, as well as angles $\theta_{\tilde{e}}$ and $\theta_{\tilde{\mu}}$ between the left and right scalar leptons of the same family. These angles describe the rotation between the sparticle and particle mass eigenbases.

Because the scalar mass splittings are required to be small, we will parametrize the scalar mass eigenvalues by the average masses, $m_{\tilde{L}}^2$ and $m_{\tilde{R}}^2$, and the mass splittings, $\delta\tilde{m}_L^2$ and $\delta\tilde{m}_R^2$. We will also keep only the leading contribution in both the mass splittings and the mixing angles. The pure nuclear physics calculations needed for $\mu - e$ conversion involve several integrals [6] and have been tabulated for currently interesing nuclei Al, Ti and Pb. We shall concentrate here on the Ti measurement, since it is at present the most accurate. Inserting: $Z_{eff} \simeq 17.6$, $F_p(q^2 = -m_\mu^2) \simeq 0.54$, $Z = 22$, $\Gamma_{capt} \simeq (2.596 \pm 0.012) \times 10^6\ s^{-1}$ and $N = 26$, we obtain, for the photonic branching ratios:

$$R_{photo} = 4.2 \times 10^{-5} \Bigg\{ \theta_L^2 \left(\frac{M_W}{m_{\tilde{L}}}\right)^4 (A_L)^2 \left(\frac{\delta\tilde{m}_L^2}{m_{\tilde{L}}^2}\right)^2$$

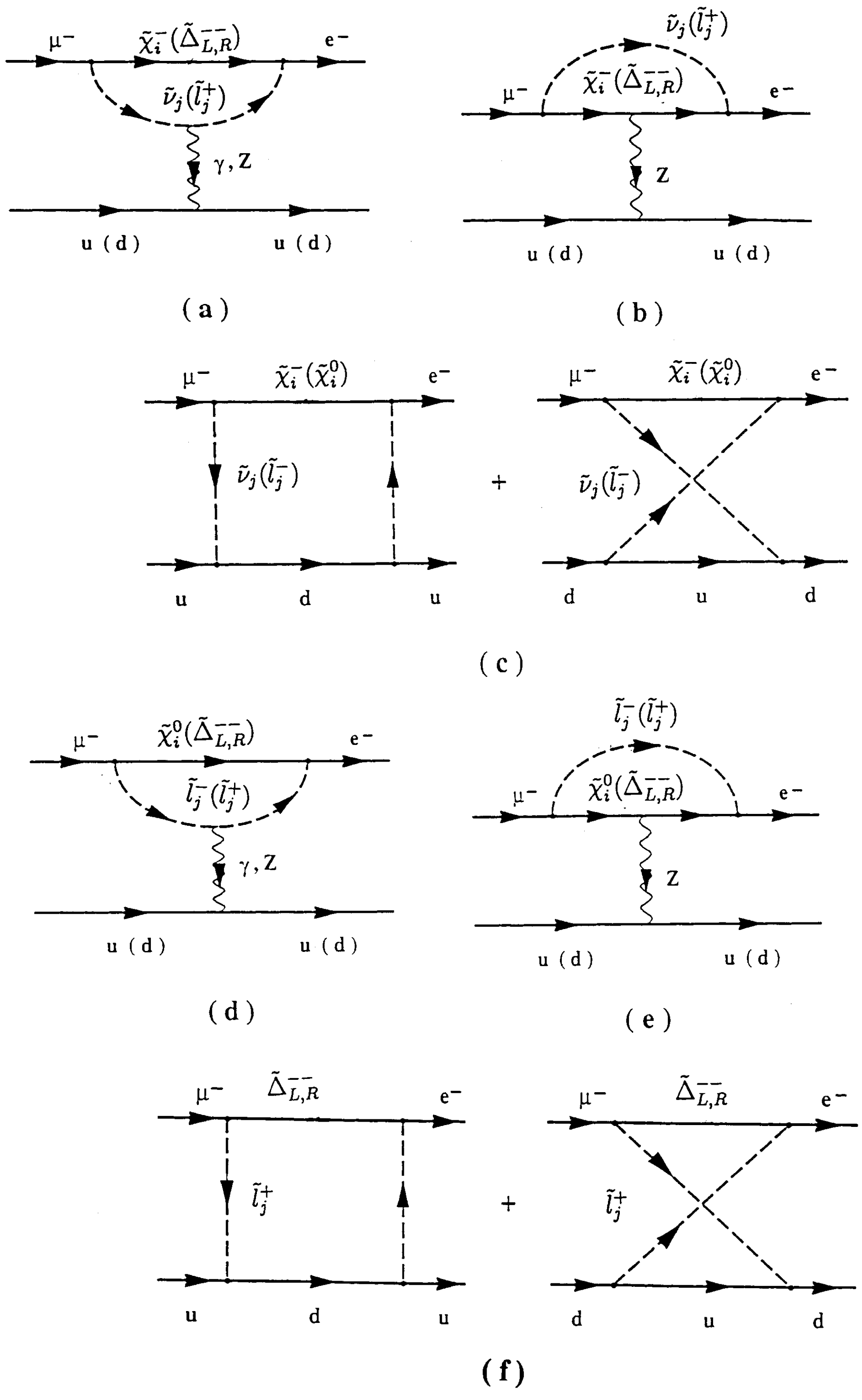

Fig. 1. One-loop contributions to the $\mu^- \to e^-$ conversion rate in nuclei in the left-right supersymmetric model. Here $\tilde{\chi}_i^{\pm}$ represents a chargino state and i runs from 1 to 8; $\tilde{\chi}_i^0$ represents a neutralino state and i runs from 1 to 11; and $\tilde{\Delta}_{L,R}^{--}$ represents a doubly charged Higgsino state

$$+\theta_R^2 \left(\frac{M_W}{m_{\tilde{R}}}\right)^4 (A_R)^2 \left(\frac{\delta \tilde{m}_R^2}{m_{\tilde{R}}^2}\right)^2 \Bigg\} \qquad (14)$$

where θ_L and θ_R represent the mixing angles between e_L and μ_L, and e_R and μ_R, respectively. We analyse in detail the matrix elements in the lepton-flavour violating conversion $\mu \to e$. We use the loop functions which are defined below. The argument of these loop functions is $r_{pk} = M_k^2/m_p^2$ where k represents the chargino, neutralino, or doubly charged Higgsino and p represents the slepton or sneutrino. The chargino and neutralino masses enter the theory via their mass eigenvalues and mixing matrices.

Following [28], we employ the following notation: the U, N matrices rotate the gaugino/Higgsino interaction basis into the neutralino/chargino mass basis. N^0 is the matrix for the neutralinos; U^+ is the matrix for the charginos $\tilde{W}_{L,R}^+$ and $\tilde{H}_u^+$; and U^- is for the charginos $\tilde{W}^-$ and $\tilde{H}_d^-$.

$U^{--}_{\Delta_{L,R}}$, $U^{++}_{\Delta_{L,R}}$ are mixing matrices for the doubly charged $\tilde{\Delta}_{L,R}$ and $\tilde{\delta}_{L,R}$ Higgsino mixing. The explicit form of these matrices is found in [28]. The functions A_L and A_R corresponding to the photon penguin graphs in Fig. 1 are given below:

$$A_L = A_{Lf} + A_{Lh} + A_{Lg} + A_{Lj} + A_{L\Delta} \tag{15}$$
$$A_R = A_{Rf} + A_{Rh} + A_{Rg} + A_{Rj} + A_{R\Delta} \tag{16}$$

Here A_L represents the left-handed contribution, and A_R the contribution from the right-handed sector. The individual contributions are as follows:

For neutralinos, left-handed fermions, with an external chirality flip:

$$A_{Lg} = \frac{1}{2}\left(N^0_{W_L k} + \frac{g_1}{g_2}N^0_{Bk}\right)^2 f^e_{ph}(r_{l_L k}) \tag{17}$$

For the neutralinos, left-handed fermions, with an internal chirality flip:

$$\begin{aligned} A_{Lh} = {} & \frac{(A+\mu\tan\beta)M_{\chi^0_k}}{m^2_{\tilde{l}_L}}\left(N^0_{W_R k} + \frac{g_1}{g_2}N^0_{Bk}\right) \\ & \times\left(N^0_{W_L k} + \frac{g_1}{g_2}N^0_{Bk}\right) f^i_{ph}(r_{l_L k}) - \frac{M_{\chi^0_k}}{\sqrt{2}g_2 v_1}\left(N^0_{Hk}\right) \\ & \times\left(N^0_{W_L k} + \frac{g_1}{g_2}N^0_{Bk}\right) f^i_{ph}(r_{l_L k}) + \left(\frac{m^4_{\tilde{l}_L}}{\delta m^4_{\tilde{\nu}_L}}\right) \\ & \times\frac{\delta A_{\bar{\mu}e}M_{\chi^0_i}}{m^2_{\tilde{l}_L} - m^2_{\tilde{l}_R}}\left(N^0_{W_R k} + \frac{g_1}{g_2}N^0_{Bk}\right) \\ & \times\left(N^0_{W_L k} + \frac{g_1}{g_2}N^0_{Bk}\right) \\ & \times\left[\frac{f^i_{ph}(r_{l_L k})}{m^2_{\tilde{l}_L}} - \frac{f^i_{ph}(r_{l_R k})}{m^2_{\tilde{l}_R}}\right] \end{aligned} \tag{18}$$

For charginos, left-handed fermions, with an external chirality flip:

$$A_{Lf} = -\left(\frac{m^4_{\tilde{l}_L}}{m^4_{\tilde{\nu}_L}}\right)\left(U^+_{W_L k}\right)^2 g^e_{ph}(r_{\nu_R k}) \tag{19}$$

For charginos, left-handed fermions, with an internal chirality flip:

$$\begin{aligned} A_{Lj} = {} & \left(\frac{m^4_{\tilde{l}}}{m^4_{\tilde{\nu}}}\right)\left(U^-_{Hk}\right)\left(U^+_{W_L k}\right) g^i_{ph}(r_{\nu_L k}) \\ & + \frac{(A+\mu\cot\beta)M_{\chi_k}}{m^2_{\tilde{l}}}\left(U^-_{W_L k}\right)\left(U^+_{W_R k}\right) g^i_{ph}(r_{\nu_L k}) \\ & + \left(\frac{m^4_{\tilde{l}_L}}{\delta m^4_{\tilde{\nu}_L}}\right)\frac{\delta A_{\bar{\nu}_\mu \nu_e}M_{\chi_k}}{m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R}}\left(U^-_{W_L k}\right)\left(U^+_{W_R k}\right) \\ & \times\left[\frac{g^i_{ph}(r_{\nu_L k})}{m^2_{\tilde{\nu}_L}} - \frac{g^i_{ph}(r_{\nu_R k})}{m^2_{\tilde{\nu}_R}}\right] \end{aligned} \tag{20}$$

For doubly charged Higgsinos, left-handed fermions, with an external chirality flip:

$$A_{L\Delta} = -\frac{M_{LR}}{2g_2 v_R}\left(U^{--}_{\Delta_R}\right)^2\left(2g^e_{ph}(r_{l_L\Delta}) - f^e_{ph}(r_{l_L\Delta})\right) \tag{21}$$

There are no contributions from doubly charged Higgsinos with internal chirality flip because the left and right doubly charged Higgsinos do not mix.

We have similar expression for the right handed contributions.

For neutralinos, right-handed fermions, with an external chirality flip:

$$A_{Rg} = \frac{1}{2}\left(N^0_{W_R k} + \frac{g_1}{g_2}N^0_{Bk}\right)^2 f^e_{ph}(r_{l_R k}) \tag{22}$$

For neutralinos, right-handed fermions, with an internal chirality flip:

$$\begin{aligned} A_{Rh} = {} & \frac{(A+\mu\tan\beta)M_{\chi^0_k}}{m^2_{\tilde{l}_R}}\left(N^0_{W_L k} + \frac{g_1}{g_2}N^0_{Bk}\right) \\ & \times\left(N^0_{W_R k} + \frac{g_1}{g_2}N^0_{Bk}\right) f^i_{ph}(r_{l_R k}) - \frac{M_{\chi^0_k}}{g_2 v_1}\left(N^0_{Hk}\right) \\ & \times\left(N^0_{W_R k} + \frac{g_1}{g_2}N^0_{Bk}\right) f^i_{ph}(r_{l_R k}) + \left(\frac{m^4_{\tilde{e_R}}}{\delta m^4_{\tilde{\nu}_R}}\right) \\ & \times\frac{\delta A_{\mu\bar{e}}M_{\chi^0_k}}{m^2_{\tilde{l}_L} - m^2_{\tilde{l}_R}}\left(N^0_{W_R k} + \frac{g_1}{g_2}N^0_{Bk}\right) \\ & \times\left(N^0_{W_L k} + \frac{g_1}{g_2}N^0_{Bk}\right) \\ & \times\left[\frac{f^i_{ph}(r_{l_L k})}{m^2_{\tilde{l}_L}} - \frac{f^i_{ph}(r_{l_R k})}{m^2_{\tilde{l}_R}}\right] \end{aligned} \tag{23}$$

For charginos, right-handed fermions, with an external chirality flip:

$$A_{Rf} = -\left(\frac{m^4_{\tilde{l}_R}}{m^4_{\tilde{\nu_R}}}\right)\left(U^+_{W_R k}\right)^2 g^e_{ph}(r_{\nu_R k}) \tag{24}$$

For charginos, right-handed fermions, with an internal chirality flip:

$$\begin{aligned} A_{Rj} = {} & \left(\frac{m^4_{\tilde{l}_R}}{m^4_{\tilde{\nu}_R}}\right)\left(U^-_{Hk}\right)\left(U^+_{W_R k}\right) g^i_{ph}(r_{\nu_R k}) \\ & + \frac{(A+\mu\cot\beta)M_{\chi_k}}{m^2_{\tilde{\nu}_R}}\left(U^-_{W_L k}\right)\left(U^+_{W_R k}\right) g^i_{ph}(r_{\nu_R k}) \\ & + \left(\frac{m^4_{\tilde{l}_R}}{\delta m^4_{\tilde{\nu}_R}}\right)\frac{\delta A_{\bar{\nu}_e \nu_\mu}M_{\chi_k}}{m^2_{\tilde{\nu}_L} - m^2_{\tilde{\nu}_R}}\left(U^-_{W_L k}\right)\left(U^+_{W_R k}\right) \\ & \times\left[\frac{g^i_{ph}(r_{\nu_L k})}{m^2_{\tilde{\nu}_L}} - \frac{g^i_{ph}(r_{\nu_R k})}{m^2_{\tilde{\nu}_R}}\right] \end{aligned} \tag{25}$$

For doubly charged Higggssinos, right-handed fermions, with an external chirality flip:

$$A_{R\Delta_f} = -\frac{M_{LR}}{2g_2 v_R}\left(U^{--}_{\Delta_R}\right)^2\left(2g^e_{ph}(r_{l_R\Delta}) - f^e_{ph}(r_{l_R\Delta})\right) \tag{26}$$

Although the equations depend very weakly on the trilinear scalar parameter A, justifying setting $A = 0$, we include the complete A dependence here. Equations (27) to (30) give definitions for the non-universality of the A terms used above.

$$\delta A_{\bar{\mu}e} = A_{\bar{\mu}e} - A_{\bar{\mu}\mu} \quad (27)$$
$$\delta A_{\bar{e}\mu} = A_{\bar{e}\mu} - A_{\bar{\mu}\mu} \quad (28)$$
$$\delta A_{\bar{\nu}_\mu \nu_e} = A_{\bar{\nu}_\mu \nu_e} - A_{\bar{\nu}_\mu \nu_\mu} \quad (29)$$
$$\delta A_{\bar{\nu}_e \nu_\mu} = A_{\bar{\nu}_e \nu_e} - A_{\bar{\nu}_e \nu_\mu} \quad (30)$$

The $\mu^- \to e^-$ loop functions are:

$$f^e_{ph}(r) = \frac{1}{6(1-r)^4}\big[13 - 60r + 111r^2 - 106r^3 - 6r^2(6r-1)\log r\big] \quad (31)$$
$$f^i_{ph}(r) = \frac{1}{(1-r)^3}\left(1 - r^2 + -2r\log r\right) \quad (32)$$
$$g^e_{ph}(r) = \frac{1}{6(1-r)^4}\big[98 - 267r + 210r^2 - 41r^3 + 6(12-17r)\log r\big] \quad (33)$$
$$g^i_{ph}(r) = \frac{1}{(1-r)^3}\left(-3 + 4r - r^2 - 2\log r\right) \quad (34)$$

4.2 The non-photonic contributions

The non-photonic currents generated by Fig. 1 (a to f) can be written as:

$$J^{(2)}_\lambda = \bar{N}\gamma_\lambda \frac{1}{2}[(3 + f_V\tau_3) + (f_V + f_A\tau_3)\gamma_5]N \quad (35)$$

for the hadronic current, and

$$j^\lambda_{(2)} = \bar{u}(p)\gamma_\lambda \frac{1}{2}(f_{\tilde{V}} + f_{\tilde{A}}\gamma_5)u(p+q) \quad (36)$$

for the leptonic current, where $f_{\tilde{V}}$, $f_{\tilde{A}}$ are model-dependent form factors. The non-photonic contributions to $\mu - e$ conversion are given by the following conversion ratio [27]:

$$R_{nph}(\mu^- N \to e^- N) = \frac{G_F^2\alpha^3 m_\mu^3 p_e E_e Z^4_{eff}F_p^2}{2\pi^2 Z\Gamma_{capt}}\Bigg[(f_V^2 + f_A^2)Q_W^2 + 2\frac{M^2_{Z_L}}{M^2_{Z_R}} \times (f_V f'_V + f_A f'_A)Q_W Q'_W + \left(\frac{M^2_{Z_L}}{M^2_{Z_R}}\right)^2 \times (f'^2_V + f'^2_A)Q'^2_W\Bigg] \quad (37)$$

where, inserting corresponding values the left-right parameters:

$$f_V = -\frac{\sqrt{2}e^2}{3G_F m_\mu^2} f_{E0}(q^2) \quad (38)$$
$$f_A = \frac{\sqrt{2}e^2}{3G_F m_\mu^2} f_{M0}(q^2) \quad (39)$$
$$f'_V = -\frac{\sqrt{2}e^2}{3G_F m_\mu^2}\frac{M^2_{Z_R}}{M^2_{Z_L}} f_{E0}(q^2) \quad (40)$$
$$f'_A = \frac{\sqrt{2}e^2}{3G_F m_\mu^2}\frac{M^2_{Z_R}}{M^2_{Z_L}} f_{M0}(q^2) \quad (41)$$
$$Q_W = (2Z+N)\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right) + (Z+2N)\left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\right) \quad (42)$$
$$Q'_W = (2Z+N)\left(\frac{1}{2} + \frac{1}{2}\tan^2\theta_W\right) + (Z+2N)\left(-\frac{1}{2} - \frac{1}{2}\tan^2\theta_W\right), \quad (43)$$

we obtain the expression for the non-photonic conversion rate due to Z_R and Z_L as:

$$R_{nph}(\mu^- N \to e^- N) = \frac{16\alpha^5 p_e E_e Z^4_{eff} F_p^2}{9m_\mu Z\Gamma_{capt}}\left[|f_{E0}|^2 + |f_{M0}|^2\right]\left\{\frac{1}{2}(Z-N) \times \left[1 - \left(\frac{M^2_{Z_L}}{M^2_{Z_R}}\right)^2 \frac{\cos 2\theta_W}{\cos^2\theta_W}\right] - 2Z\sin^2\theta_W\right\}^2 \quad (44)$$

Note that the non-photonic rate is proportional to Z^4_{eff}/Z (to be compared with $Z^4_{eff}Z$ for the photonic rate). Using the same values for the nuclear structure as in the previous subsection, this expression translates into the following conversion ratio for Ti, expressed as a sum of chargino, neutralino and doubly charged Higgsinos contributions:

$$R_{nph}(\mu^- \to e^-) = \frac{16 p_e E_e \alpha^5 Z^4_{eff} F_p^2}{9m_\mu Z\Gamma_{capt}}\left\{\theta_L^2\left(\frac{M_W}{m_{\tilde{L}}}\right)^4 (B_L)^2\left(\frac{\delta\tilde{m}^2_L}{m^2_{\tilde{L}}}\right)^2 + \theta_R^2\left(\frac{M_W}{m_{\tilde{R}}}\right)^4 (B_R)^2\left(\frac{\delta\tilde{m}^2_R}{m^2_{\tilde{R}}}\right)^2\right\}\left\{\frac{1}{2}(Z-N) \times \left[1 - \left(\frac{M^2_{Z_L}}{M^2_{Z_R}}\right)^2\frac{\cos 2\theta_W}{\cos^2\theta_W}\right] - 2Z\sin^2\theta_W\right\}^2 \quad (45)$$

where

$$B_L = B_{Lf} + B_{Lg} + B_{L\Delta} \quad (46)$$
$$B_R = B_{Rf} + B_{Rg} + B_{R\Delta} \quad (47)$$

Here B_L represents the left-handed contribution, and B_R the contribution from the right-handed sector. The individual contributions are given below (note that only external chirality flip terms contribute to the non-photonic rates of conversion).

For neutralinos, left-handed fermions:

$$B_{Lg} = \frac{1}{2}\left(N^0_{W_L k} + \frac{g_1}{g_2}N^0_{Bk}\right)^2 f_{nph}(r_{l_L k}) \quad (48)$$

For charginos, left-handed fermions:

$$B_{Lf} = -\left(\frac{m^4_{\tilde{l}_L}}{m^4_{\tilde{\nu}_L}}\right)\left(U^+_{W_L k}\right)^2 g_{nph}(r_{\nu_R k}) \quad (49)$$

For doubly charged Higgsinos, left-handed fermions:

$$B_{L\Delta} = -\frac{M_{LR}}{2g_2 v_R}\left(U^{--}_{\Delta_R}\right)^2 \left(2g_{nph}(r_{l_L\Delta}) - f_{nph}(r_{l_L\Delta})\right) \quad (50)$$

There are similar expression for the right handed contributions.

For neutralinos, right-handed fermions:

$$B_{Rg} = \frac{1}{2}\left(N^0_{W_R k} + \frac{g_1}{g_2} N^0_{Bk}\right)^2 f_{nph}(r_{l_R k}) \quad (51)$$

For charginos, right-handed fermions:

$$B_{Rf} = -\left(\frac{m^4_{\tilde{l}_R}}{m^4_{\tilde{\nu}_R}}\right)\left(U^+_{W_R k}\right)^2 g_{nph}(r_{\nu_R k}) \quad (52)$$

For doubly charged Higgssinos, right-handed fermions:

$$B_{R\Delta} = -\frac{M_{LR}}{2g_2 v_R}\left(U^{--}_{\Delta_R}\right)^2 \left(2g_{nph}(r_{l_R\Delta}) - f_{nph}(r_{l_R\Delta})\right) \quad (53)$$

Finally, we give the expressions for the $\mu^- \to e^-$ non-photonic loop functions:

$$f_{nph}(r) = \frac{1}{(1-r)^4} \times \left(2 - 9r + 18r^2 - 18r^3 + 6r^3 \log r\right) \quad (54)$$

$$g_{nph}(r) = \frac{1}{(1-r)^4}\left[16 - 45r + 36r^2 - 7r^3 + 6r(2-3r)\log r\right] \quad (55)$$

5 Numerical results and discussion

Having introduced the theoretical framework for $\mu - e$ conversion, we turn to the quantitative analysis of this process in terms of the parametrs of the left right model. The flavour violating decays are sensitive to the universal GUT parameters m_0 (the scalar mass), the trilinear coupling A, the value and the sign of the Higgs mixing parameter μ, the value of $\tan\beta$, the values of the left- and right-handed gaugino masses M_L and M_R, and M_{LR}. We fix here the value of $A_0 = m_0$, but allow M_L, μ, $\tan\beta$ and M_R to vary within the experimentally allowed parameter region. We set $M_{LR} = 100\ GeV$ to maximize the doubly charged Higgsino contribution, but note that this value does not affect strongly the conversion rate. In addition, the flavour violating decays will depend on the neutrino mixings which are assummed to be large enough to explain the neutrino data [1]. We briefly review some popular realizations of neutrino mixings.

The neutrino mix through $\nu_\alpha = K_{\alpha i}\nu_i$, where α denotes the flavour e, μ, τ and i denotes mass eigenvalues 1,2,3. If we assume that the atmospheric data is fit by $\sin^2 2\theta_{atm} = 1$, the mixing matrix can be parametrized as:

$$K_{\alpha i} = \begin{pmatrix} c & -s & K_{e3} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (56)$$

In the above formula $s = \sin\theta_{sol}$, $c = \cos\theta_{sol}$. (If $\sin^2 2\theta_{atm} = 8/9$ the matrix is similar, indeed we need not consider it separately). In this pattern the solar neutrino deficit is explained either by the small mixing angle MSW (oscillation enhanced by the solar core) solution (MSW-SA): $\sin\theta_{sol} \approx 5.5 \times 10^{-3}$, $\sin\theta_{e\tau}$, $\sin\theta_{e\mu}$ small; or by the large-mixing angle of the MSW solution (MSW-LA): $\sin^2 2\theta_{sol} \approx 1$. The other solution is for oscillations without help from the solar core, called vacuum oscillations (or the just-so solution) and it requires (VO): $\sin^2 2\theta_{sol} \approx 0.75$. If one includes the LSND data, one might need to introduce a sterile neutrino as the fourth neutrino. There are cogent arguments that the only schemes consistent with all the experiments are those which include two pairs of neutrinos with nearly degenerate masses separated by a gap of order 1 eV. This scenario has a 4×4 neutrino mass matrix in which the pairs $\nu_e - \nu_s$ and $\nu_\mu - \nu_\tau$ mix maximally within doublets but mixing between doublets is weak; scenarios with more than one sterile neutrino also exist, but we will not discuss them here [29]. This solution (LSND) requires $\sin^2 2\theta_{sol} \approx 0.003 - 0.03$. In all cases K_{e3} is very small and constrained by the CHOOZ data to be $|K_{e3}| \leq 0.2$ [30]. The restrictions on mass splittings coming from these mixings are [29]:

$$\begin{aligned}(\Delta m^2)_{MSW-LA} &\sim 3.5 \times 10^{-3}\ eV^2 \\ (\Delta m^2)_{MSW-SA} &\sim 5 \times 10^{-6}\ eV^2 \\ (\Delta m^2)_{VO} &\sim 10^{-10}\ eV^2 \\ (\Delta m^2)_{LSND} &\sim 0.2 - 2\ eV^2\end{aligned} \quad (57)$$

The limits on neutrino masses come from several considerations. First, there are the direct bounds: $m_{\nu_e} \leq 5\ eV$, $m_{\nu_\mu} \leq 170 KeV$ and $m_{\nu_\tau} \leq 18\ MeV$. However, cosmological restrictions coming from the critical density for neutrino hot dark matter impose neutrino masses whose sum cannot exceed a few eV, $\sum \nu_i \leq 6\ eV$. This would allow the heaviest neutrinos to have masses of order 2 eV, if all three neutrinos are degenerate in mass. Most theoretical fits to the data indicate however a tau neutrino of mass 1 eV [29].

Clearly, from the point of view of muon decays, the most interesting scenario is either the MSW-LA or VO scenarios; these allow for significant mixing between $e - \mu$ sector. If the dominant mixing is MSW-SA, one would expect the most pronounced lepton flavour violation to occur in the $\tau - \mu$ sector and therefore be visible in a large, possible observable, $\tau \to \mu\gamma$ branching ratio. As an estimate for the mixing angle we shall take the usual assumption that each is equal to the square root of the masses of the two particles it relates: for the leptons, $\theta_L = \theta_R = \sqrt{e/\mu}$. This result is inspired by GUT theories, quark-lepton universality, and the successful quark

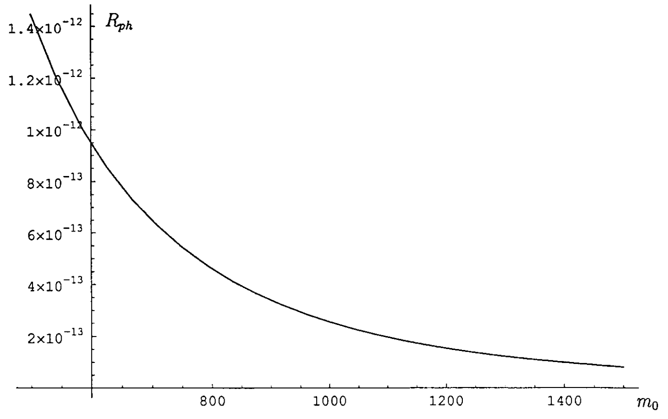

Fig. 2. Variation of the photonic conversion rate with the universal scalar mass m_0. The other parameters are set as $M_L = 200\ GeV,\ \mu = 200\ GeV,\ M_R = 10\ TeV$ and $\tan\beta = 3$

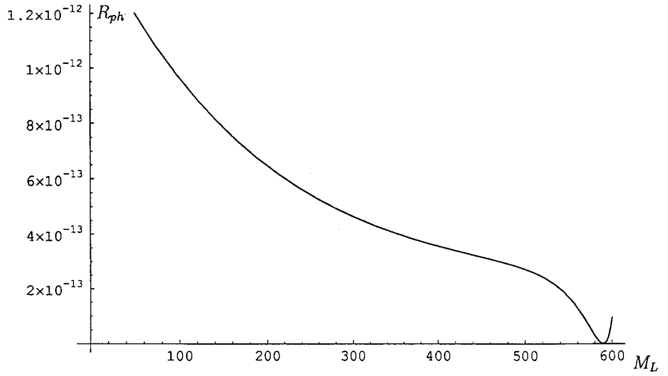

Fig. 3. Variation of the photonic conversion rate with the left-handed gaugino mass M_L. The other parameters are set as $m_0 = 700\ GeV,\ \mu = 200\ GeV,\ M_R = 10\ TeV$ and $\tan\beta = 3$

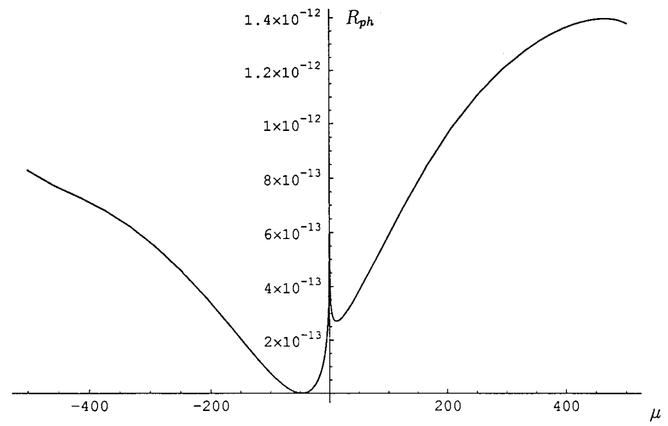

Fig. 4. Variation of the photonic conversion rate with the Higgs mixing parameter μ, for both positive and negative μ values. The other parameters are set as $M_L = 100\ GeV,\ m_0 = 700\ GeV,\ M_R = 10\ TeV$ and $\tan\beta = 3$

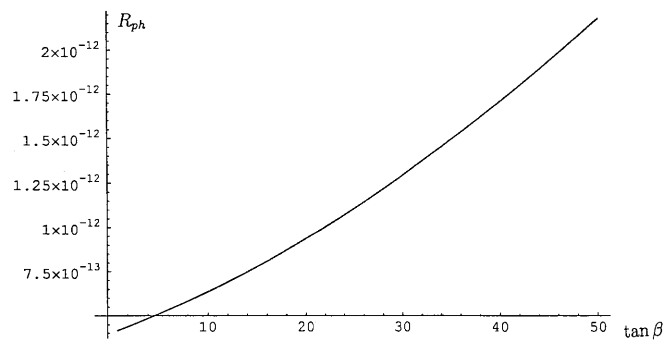

Fig. 5. Variation of the photonic conversion rate with the $\tan\beta = \kappa_d/\kappa_u$. The other parameters are set as $M_L = 200\ GeV,\ \mu = 200\ GeV,\ M_R = 10\ TeV$ and $m_0 = 800\ GeV$

counterpart form: $\theta_{Cabbibo} = \sqrt{d/s}$. However, we will also consider violations of this ratio coming from the Dirac neutrino mass-induced mixing, as discussed in Sect. 3.

We first shall present the analysis for the photonic conversion rate. Since the photonic rate varies extremely weakly with the right-handed scale M_R, we shall keep thoughtout these plots M_R constant at $M_R = 10\ TeV$. The non-photonic rate is usually less that 10^{-3} the value of the photonic rate and can be safely neglected for restrictions on SUSY parameters, with the exception of the right-handed mass and mixing, as we shall see later.

Figures 2–5 show the dependence of the conversion ratio with the parameters of the LR SUSY model. We first plot the photonic conversion rate as a function of the universal scalar mass m_0 for a light gaugino-Higgsino scenario and low $\tan\beta$, $M_L = 100\ GeV,\ \mu = 200\ GeV$ and $\tan\beta = 3$ (Fig. 2). Under these circumstances, the present data restricts scalar masses to be heavier than 750 GeV. We then plot the variation of the photonic conversion with the scale of the left-handed gaugino mass, M_L. We keep $\mu = 200\ GeV$ and $\tan\beta = 3$, and set $m_0 = 700\ GeV$ (Fig. 3). The present bound on the conversion rate is saturated at $M_L \simeq 250\ GeV$. In Fig. 4 we plot the dependence of the conversion rate on the Higgsino mixing parameter μ, for both positive and negative values of $sign(\mu)$, for $M_L = 100\ GeV,\ m_0 = 700\ GeV$ and $\tan\beta = 3$. The present bound is saturated faster for positive μ, requiring $\mu \geq 175\ GeV$, than for negative μ, where the requirement is $|\mu| \geq 350$. Neither of these are too restrictive. The dependence of the conversion rate on $\tan\beta$ is shown in Fig. 5 for $M_L = \mu = 200\ GeV, m_0 = 800\ GeV$. As in the case of the decay $\mu \to e\gamma$, the conversion rate is shown to increase rapidly with $\tan\beta$ and the process becomes quickly ruled out for a light gaugino/Higgsino sector for $\tan\beta$ as low as 10.

Figures 6–8 show the dependence of the conversion ratio with the universal scalar parameter m_0 for $M_L = 100\ GeV$ (dashed curve) and 500 GeV (solid curve) in Fig. 6; for $\mu = 500\ GeV$ (dashed curve) and $-500\ GeV$ (solid curve) in Fig. 7; and for $\tan\beta = 3$ (dashed curve) and 25 (solid curve) in Fig. 8. Figure 9 shows the restrictions coming from the conversion rate on the $M_L - \mu$ parameter space for the universal scalar mass $m_0 = 800\ GeV$. Even at present experimental sensitivity, regions of light gaugino-Higgsino masses are excluded.

We then analyse the dependence of the conversion rate on the neutrino mixing schemes described at the beginning of this section. The MSW-LA and VO schemes are

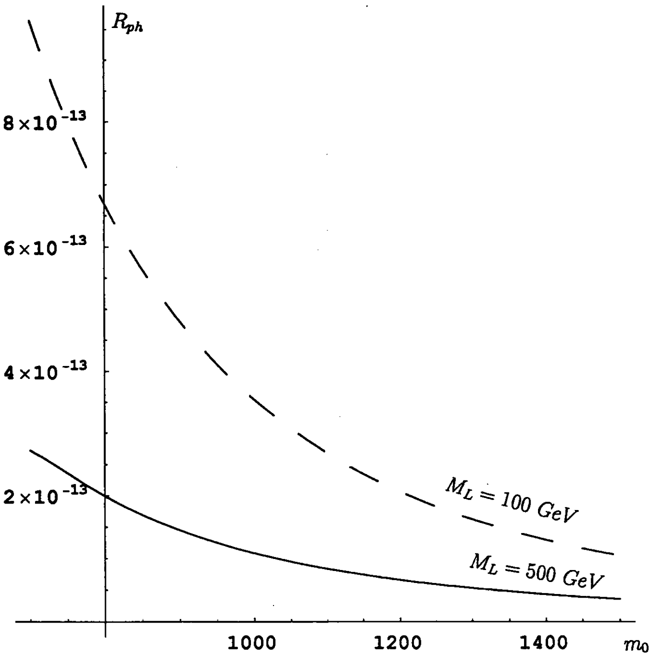

Fig. 6. Variation of the photonic conversion rate with the universal scalar mass m_0 for light $M_L = 100\ GeV$ (dashed curve) and intermediate $M_L = 500\ GeV$ (solid curve) left-handed gaugino mass scale. The other parameters are set as $\mu = 200\ GeV,\ M_R = 10\ TeV$ and $\tan\beta = 3$

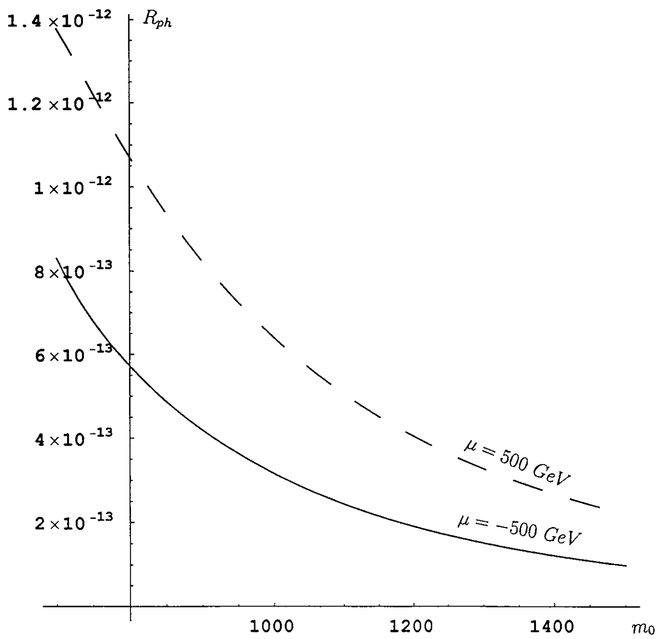

Fig. 7. Variation of the photonic conversion rate with the universal scalar mass m_0 for positive $\mu = 500\ GeV$ (dashed curve) and negative $\mu = -500\ GeV$ (solid curve) Higgs mixing parameter. The other parameters are set as $M_L = 100\ GeV,\ M_R = 10\ TeV$ and $\tan\beta = 3$

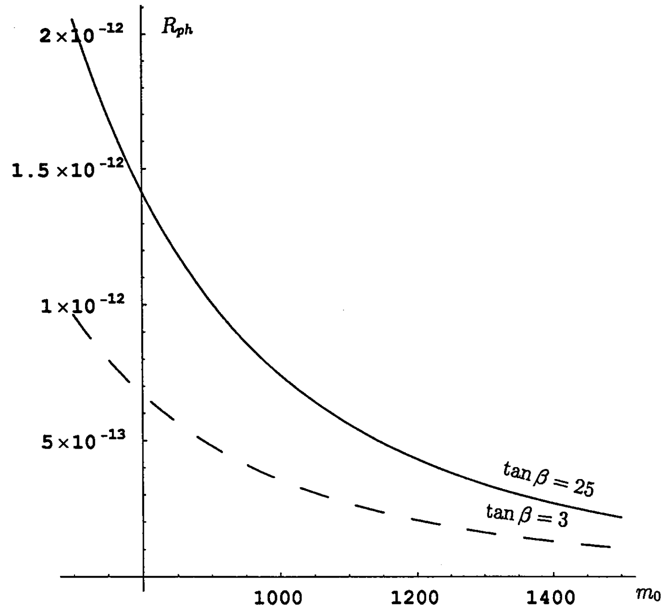

Fig. 8. Variation of the photonic conversion rate with the universal scalar mass m_0 for small $\tan\beta = 3$ (dashed curve) and intermediate-large $\tan\beta = 25$ (solid curve). The other parameters are set as $M_L = 100\ GeV,\ \mu = 200\ GeV,\ M_R = 10\ TeV$

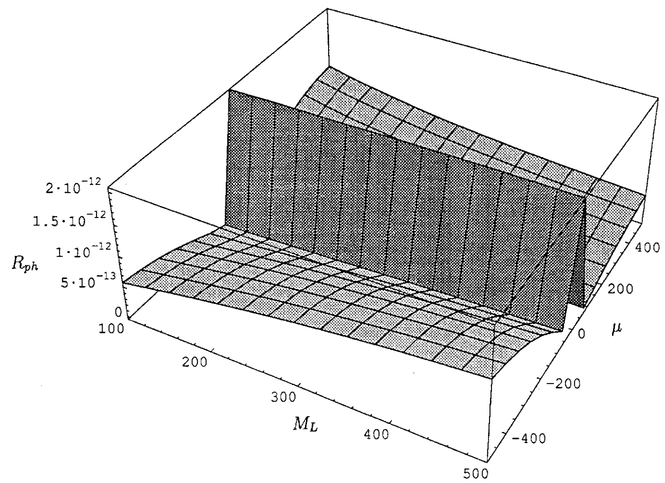

Fig. 9. Plot of the photonic conversion rate variation with the left-handed gaugino mass M_L and the Higgs mass mixing parameter μ for $m_0 = 800\ GeV$, for $M_R = 10\ TeV$ and $\tan\beta = 3$

quite close and cannot be distinguished through $\mu - e$ conversion. However, the MSW-SA mixing scheme gives significantly smaller conversion rates and, either through improved precision measurements, or in regions of light scalar masses, can be distinguished from the other two. In that case, the smuon-selectron mixing is small, the smuon-stau mixing is enhanced, and we would expect $\tau - \mu$ flavour violation to dominate considerably over $\mu - e$ flavour violation. This dependence shown in Fig. 10.

If the form factors $f_{E1}(q^2)$ and $f_{M1}(q^2)$ dominate, the branching ratios of $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^-e^+$ and $\mu^- - e^-$

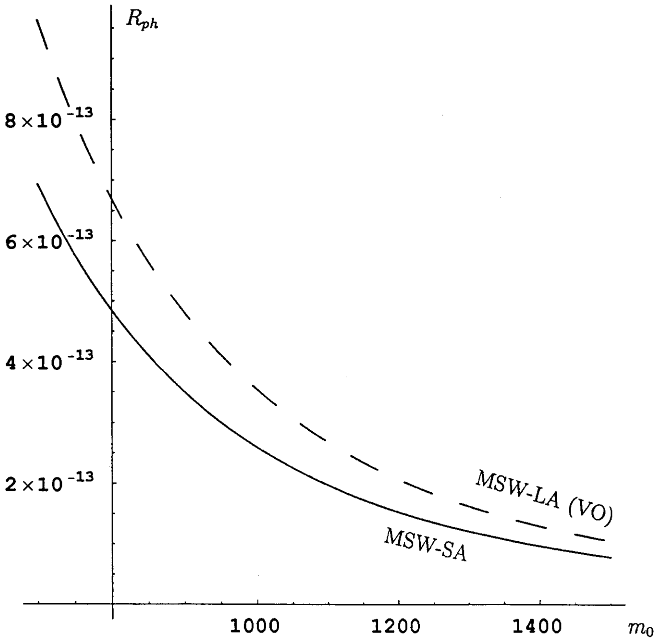

Fig. 10. Variation of the photonic conversion rate with the universal scalar mass m_0 for large $\nu_e - \nu_\mu$ mixing (MSW-LA, VO; dashed curve) and small $\nu_e - \nu_\mu$ mixing (MSW-SA; solid curve). The other parameters are set as $M_L = 100\ GeV$, $\mu = 200\ GeV$, $M_R = 10\ TeV$ and $\tan\beta = 3$

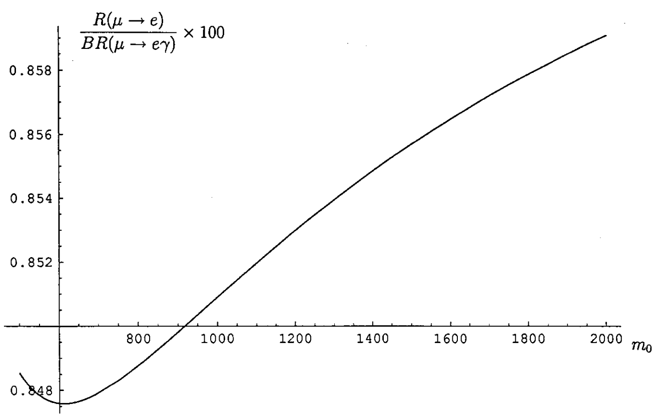

Fig. 11. Variation of the ratio of photonic conversion rate to the branching ratio of radiative muon decay, $R_{photo}(\mu \to e)/BR(\mu \to e\gamma)$, with the universal scalar mass m_0, for $M_L = 200\ GeV$, $\mu = 200\ GeV$, $M_R = 10\ TeV$ and $\tan\beta = 3$

conversion are approximately related and we expect:

$$\frac{\Gamma(\mu Ti \to eTi)}{\Gamma(\mu Ti \to capture)} \simeq \frac{1}{200} BR(\mu \to e\gamma), \qquad (58)$$

$$BR(\mu^+ \to e^+e^+e^-) \simeq \frac{1}{160} BR(\mu^+ \to e^+\gamma) \qquad (59)$$

Though the second rate is practically universal over the whole parameter spectrum, the first is not so. We plot the ratio of the conversion rate to the radiative $\mu \to e\gamma$ rate as a function of the universal scalar mass m_0 in Fig. 11. The conversion rate is somewhat enhanced compared with the naive expectation (58). We see that, even if both muon

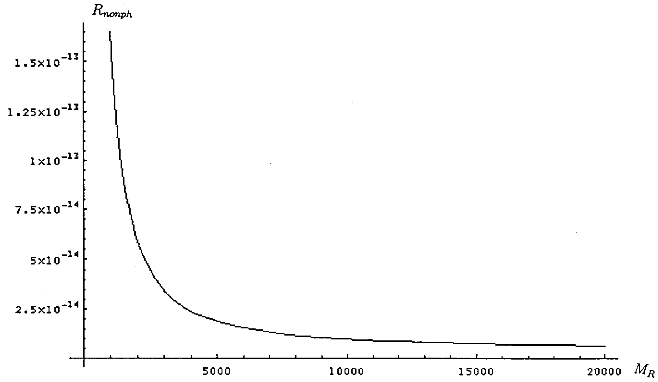

Fig. 12. Variation of the non-photonic conversion rate with the gaugino right-handed mass scale M_R. The other parameters are set as $M_L = 200\ GeV$, $\mu = 200\ GeV$, $m_0 = 500\ GeV$ and $\tan\beta = 3$

flavour violations can occur, the ratio is not universal and the conversion rate is relatively more important for regions with larger values of m_0.

Finally, we analyse the non-photonic ratio as a function of the right handed scale M_R in Fig. 12. The non-photonic conversion ratio depends sensitively on M_R and can give significant contributions for as low a right-handed scale as 5 TeV. This translates, in the exact supersymmetric limit, where the gaugino and gauge scales are approximately equal, into a visible Z_R presence in the 1-5 TeV range. It is interesting to note that at such high masses, the $Z_R - Z_L$ mixing angle is extremely small, and the ratio $(\frac{M^2_{Z_L}}{M^2_{Z_R}})^2$ in (45) gives negligible contributions, but the M_R dependence comes from the loop functions.

6 Conclusion

We have presented a complete analysis of the conversion ratio for the process $\mu \to e$ in the left-right supersymmetric model. The conversion ratio is enhanced relative to the corresponding one in MSSM. This enhancement is due to the contribution of right-handed particles in the both the gauge and matter sectors, and to a richer gaugino/Higgsino sector. The conversion ratio restricts universal scalar masses to be heavy (for a light gaugino-Higgsino spectrum), of order of 700 GeV or more. It shows extreme sensitivity to large values of $\tan\beta$ and requires that either the left-handed scale be heavy, or the sfermion masses be heavy. It allows both positive or negative values for μ, the Higgsino mixing parameter, although it saturates its present experimental bound faster for $\mu > 0$. With improved accuracy, it can distinguish between various neutrino mixing schemes. The conversion rate also has some advantages over the radiative muon decay: it is more sensitive to the right-handed scale, therefore to the presence of the Z_R, more sensitive to the presence of a doubly-charged Higgsino [8], and the ratio of the two processes is not universal. Analyses of both processes are essential for understanding lepton flavour violation, especially muon-electron mixing. In LR SUSY the lepton flavour violation is allowed to differ from the flavour violation in the quark sector through some diagrams that contribute only in the lepton sector (the diagrams with doubly charged Higgsinos and Higgs) and as such one entertains the possibility of un-related sources of flavour violation in squark and slepton mass matrices. (The latter occurs in the case where LR SUSY is *not* imbedded into a SUSY GUT structure, such as SO(10).)

The contribution of $\mu \to e$ might provide a more sensitive test for lepton flavour violation and for LR SUSY than $\mu \to e\gamma$, especially given the fact that the present experimental limit may be improved at the planned MECO experiment at Brookhaven using ^{27}Al targets, and the expected sensitivity on $R_{\mu e^-}$ is [31]:

$$R_{\mu e^-} < 2 \times 10^{-17} \quad \text{for } ^{27}Al \text{ target}, \qquad (60)$$

which implies an improvement over the existing limits of about four orders of magnitude. If this happens, whatever the mechanisms considered for $\mu - e$ conversion, it can be expected that this process will become the principal test of muon number conservation.

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